STA 623 — Fall 2013 — Dr. Charnigo

In-Class Assessment

This assessment is a strictly individual activity. Textbooks, notes, calculators, computers, and technology with Internet access are prohibited. Record what you want graded in the blue book.

[20] 1. Let F be a real-valued function of a real variable.

[10] a. Under what conditions is F the cumulative distribution function of a discrete random variable ?

[10] b. Suppose that F is, in fact, the cumulative distribution function of a discrete random variable. Consider the following statement: "The composition $\{F \circ F\}$ defined by $\{F \circ F\}(x) := F[F(x)]$ is also the cumulative distribution function of a discrete random variable." Is the statement true or false ? If true, provide a proof. If false, furnish a counterexample.

[40] 2. Let f be a real-valued function of a real variable.

[10] a. Under what conditions is f the probability density function of a continuous random variable ?

[10] b. Suppose that f is defined by $f(x) := Cx^{-2}1_{|x|\geq 1}$, where C is a constant. What choice of C yields a probability density function ?

[10] c. Let X have the probability density function in part b. Show that $E[X]$ does not exist finitely.

[10] d. Let X have the probability density function in part b. Show that $E[\cos(tX)]$ *does* exist finitely for any $t \in \mathbb{R}$. *Hint*: You are not asked to evaluate $E[\cos(tX)]$, just establish finiteness.

[40] 3. Suppose that X is a binomial random variable with parameters $n \in \{1, 2, \ldots\}$ and $p \in (0, 1)$. Suppose that Y has the same distribution as X but that Y is independent of X. Put $U := X$ and $V := X + Y$.

[10] a. Find the joint probability mass function of U and V. *Hint*: The bivariate transformation formula is *not* applicable.

[10] b. Find the marginal probability mass function of V. *Hint*: A difficult summation can be handled through multiplication and division by $\binom{2n}{v}$.

[10] c. Find the conditional probability mass function of U given that $V = v \in \{0, 1, \ldots, 2n\}$.

[10] d. Find the conditional probability mass function of V given that $U = u \in \{0, 1, \ldots, n\}$.