STA 623 — Fall 2013 — Dr. Charnigo

Written Assignment 1

Written Assignment 1 is due on Thursday 26 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer. In what follows, all sets are assumed to be events that belong to the sigma field.

[20] 1. Give an English language description for $\cap_{j=1}^{\infty} \cup_{n=j}^{\infty} \cap_{i=n}^{\infty} A_i$ like I did for $\cup_{n=1}^{\infty} \cap_{i=n}^{\infty} A_i$ in my script for Section 1.1. Justify your answer. (You may quote without proof statements from my script for Section 1.1.)

[20] 2. Show that $P(A) \geq \sum_{i=1}^{\infty} P(A \cap C_i)$ when $C_1, C_2, \ldots$ are mutually exclusive. Result 11 in my script for Section 1.2 implies that equality must hold when $C_1, C_2, \ldots$ are also collectively exhaustive, but give two separate examples to illustrate that equality may or may not hold when $C_1, C_2, \ldots$ are not collectively exhaustive.

[20] 3. You and I are both dealt 5 cards from a well-shuffled standard 52-card deck. What is the probability that both of us will receive a flush ? (This refers to all five cards having the same suit.)

[20] 4. For what values of C_1 and C_2 is $f(x) := \log x$ for $C_1 < x < C_2$ a valid probability density function ? Use R or other software of your choice to graph these values in a $C_1 C_2$-plane.

[20] 5. Prove or disprove the following claim: If X is a discrete random variable such that $\{x \in \mathbb{R} : f(x) > 0\}$ is infinite, then $F(x) < 1$ for all $x \in \mathbb{R}$.

Exercises 4 and 5 from Written Assignment 1 of STA 524 may also interest you, but they are not required of you and will not be graded.