STA 623 — Fall 2013 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Thursday 10 October at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[30] 1. Decide whether each assertion below is true or false. If true, prove the assertion. If false, furnish a counterexample.

[10] a. Let X be a random variable whose probability density function $f_X(x)$ is continuous on its support $\mathcal{X} = (0, \infty)$. Suppose there exist constants $p > 2$, $q > 0$, and $r > 0$ such that $f_X(x) \leq r$ and $\lim_{x\to\infty} x^{-p}/f_X(x) = q$. Then $E[X^{p-2}]$ exists finitely.

[10] b. Let X be a random variable whose probability density function $f_X(x)$ is continuous on its support $\mathcal{X} = (0, \infty)$. Suppose there exist constants $p > 2$ and $q > 0$ such that $\lim_{x\to\infty} x^{-p}/f_X(x) = q$. Then $E[X^{p-2}]$ exists finitely.

[10] c. Let X be a random variable whose probability density function $f_X(x)$ is continuous on its support $\mathcal{X} = (0, \infty)$. Suppose there exist constants $p > 2$ and $q > 0$ such that $\lim_{x\to\infty} x^{-p}/f_X(x) = q$. If p is an integer, then $E[X^{p-2}] = \frac{d^{p-2}}{dt^{p-2}} M_X(t)|_{t=0}$.

[70] 2. Let X have probability density function $f_X(x) := \lambda^2 x \exp[-\lambda x]$ for $x \in (0, \infty)$, where λ is a positive constant. The corresponding cumulative distribution function is $F_X(x) := 1 - \exp[-\lambda x](1 + \lambda x)$ for $x \in (0, \infty)$.

[10] a. Calculate $E[X]$ using the definitional formula $\int_0^\infty x f_X(x)\, dx$ and explicitly carrying out integration by parts.

[10] b. Calculate $E[X]$ using the definitional formula $\int_0^\infty x f_X(x)\, dx$ and appealing to the kernel method.

[10] c. Calculate $E[X]$ using the computational formula $\int_0^\infty [1 - F(x)]\, dx$. (You may assume without proof that the computational formula is applicable.)

[10] d. Calculate $E[X]$ by differentiating the moment generating function. (You may quote without proof the moment generating function provided in your textbook.)

[10] e. Put $Y := \log[X]$. Find the cumulative distribution function of Y.

[10] f. Let Y be as in part e. Find the probability density function of Y by differentiating its cumulative distribution function.

[10] g. Let Y be as in part e. Find the probability density function of Y by applying the monotone transformation formula to $f_X(x)$.