STA 623 — Fall 2013 — Dr. Charnigo

Written Assignment 3

Written Assignment 3 is due on Thursday 07 November at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[30] 1. Consider a mixture of two beta distributions with probability density function

$$f(x;\gamma,\alpha_1,\alpha_2,\beta_1,\beta_2) := (1-\gamma)\frac{\Gamma[\alpha_1+\beta_1]}{\Gamma[\alpha_1]\Gamma[\beta_1]}x^{\alpha_1-1}(1-x)^{\beta_1-1} + \gamma\frac{\Gamma[\alpha_2+\beta_2]}{\Gamma[\alpha_2]\Gamma[\beta_2]}x^{\alpha_2-1}(1-x)^{\beta_2-1}$$

for $x \in (0,1)$, where $\gamma \in [0,1]$ and $\alpha_1,\alpha_2,\beta_1,\beta_2 \in (0,\infty)$. Let $\theta := (\gamma,\alpha_1,\alpha_2,\beta_1,\beta_2)^T$.

[10] a. Show that θ is not identifiable.

[10] b. Find θ such that $\gamma(1-\gamma)(\alpha_1-\alpha_2)(\beta_1-\beta_2) \neq 0$ but $f(x;\gamma,\alpha_1,\alpha_2,\beta_1,\beta_2)$ simplifies to a uniform density. (The implication is that a non-trivial mixture of two different beta distributions can simplify to an ordinary beta distribution, which is a rather pathological feature of the two-parameter beta family.)

[10] c. Show that θ is identifiable with the restrictions $\gamma(1-\gamma)(\alpha_1-\alpha_2) > 0$ and $\beta_1 = \beta_2 = 1$.

[70] 2. Let X have the probability density function with kernel $\exp[-\tau x^p]1_{\{x>0\}}$, where $p > 0$ and $\tau > 0$.

[10] a. Find the normalizing constant for the probability density function of X.

[10] b. Show that the family of probability density functions indexed by p and τ is neither a location-scale family nor an exponential family.

[10] c. Show that, for any fixed p, the subfamily of probability density functions indexed by τ is both a scale family and an exponential family.

[10] d. Show that, if $p > 1$, then the moment generating function exists finitely for all t.

[20] e. Show that, if $p < 1$, then the moment generating function does not exist finitely for any $t > 0$ but all of the moments still exist.

[10] f. What commonly encountered parametric family is obtained when $p = 1$?