STA 623 — Fall 2013 — Dr. Charnigo

Written Assignment 4

Written Assignment 4 is due on Thursday 21 November at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[20] 1. Consider the joint probability density function $f_{X,Y}(x, y) := 8xy$ for $0 < x < y < 1$. In class we found the corresponding marginal probability density functions $f_X(x)$ and $f_Y(y)$.

[10] a. Calculate $f_X(x)f_Y(y)$ and compare to $f_{X,Y}(x, y)$. What do you conclude ?

[10] b. On the other hand, $8xy$ is clearly the product of a function of x with a function of y. Does this contradict part a ?

[10] 2. Let X have probability density function $f_X(x) := \lambda \exp(-\lambda x)1_{x>0}$, where $\lambda > 0$. Calculate $M_X(t)$ for $t \in (-\lambda, \lambda)$.

[10] 3. Let W have probability density function $f_W(w) := (\lambda/2) \exp(-\lambda|w|)$, where $\lambda > 0$. Calculate $M_W(t)$ for $t \in (-\lambda, \lambda)$.

[30] 4. Let X and Y be independent exponential random variables, each with mean $1/\lambda$. What is the distribution of $X - Y$?

[10] a. Answer by finding $M_{X-Y}(t)$ for $t \in (-\lambda, \lambda)$.

[20] b. Answer by applying the bivariate transformation formula with $U := X - Y$ and $V := X + Y$.

[30] 5. Let X and Y be independent exponential random variables, each with mean $1/\lambda$.

[10] a. Find the joint probability density function of X and $X + Y$. (If you like, you may introduce another symbol, such as V, for $X + Y$.)

[10] b. Find the marginal probability density function of $X + Y$. To what parametric family does this distribution belong ?

[10] c. Find the conditional probability density function of X given that $X + Y = c$, where c is some positive constant. To what parametric family does this distribution belong ?