

## Additional simulation results

The tables below are expanded versions of Tables 1 through 4 in the manuscript and present more detailed results for the simulation study described in Section 6.1 of the manuscript. The Strategy rows  $CE_{h;\beta_n;M_n}$  pertain to compound estimation with bandwidth  $h$  for the local regression pointwise estimators, parameter  $\beta_n$  in expression (4) of the manuscript, and  $3^{M_n}$  centering points. The rows  $LR_h$  pertain to local regression with bandwidth  $h$ , while the rows  $SS_{df}$  pertain to spline smoothing with degrees of freedom  $df$ . Any Strategy row with subscript  $GC_p$  refers to the tuning parameter(s) selected by the  $GC_p$  criterion, which were not necessarily the same for each of the 100 data sets. The columns 0 through 6 present the average value over the 100 data sets of expression (17) of the manuscript for  $0 \leq j \leq 6$ , while the column RSD presents the average value of expression (18) of the manuscript. The column  $GC_p$  votes identifies the number of the 100 data sets for which  $GC_p$  selected that row's tuning parameter(s).

Table 1: Recovering derivatives of the oscillatory test function ( $n = 125$ ).

Strategy	0	1	2	3	4	5	6	RSD	$GC_p$ votes
$CE_{0.35;10;2}$	0.663	0.845	0.796	0.732	0.480	-0.264	-2.655	6.403	0
$CE_{0.40;10;2}$	0.938	0.913	0.879	0.851	0.778	0.541	-0.158	2.257	0
$CE_{0.45;10;2}$	0.943	0.907	0.865	0.842	0.778	0.667	0.313	1.684	0
$CE_{0.35;15;2}$	0.959	0.932	0.871	0.695	-0.025	-3.717	-25.081	32.367	0
$CE_{0.40;15;2}$	0.960	0.935	0.902	0.846	0.686	-0.026	-3.268	5.964	0
$CE_{0.45;15;2}$	0.949	0.914	0.872	0.839	0.744	0.455	-0.682	2.909	0
$CE_{0.35;20;2}$	0.970	0.943	0.856	0.443	-2.351	-23.310	-190.076	219.524	0
$CE_{0.40;20;2}$	0.964	0.941	0.904	0.813	0.412	-2.089	-18.808	23.864	0
$CE_{0.45;20;2}$	0.950	0.915	0.871	0.825	0.662	-0.049	-3.822	6.648	0
$CE_{0.35;10;3}$	0.959	0.945	0.915	0.899	0.887	0.850	0.765	0.780	0
$CE_{0.40;10;3}$	0.956	0.934	0.896	0.888	0.871	0.848	0.802	0.804	0
$CE_{0.45;10;3}$	0.951	0.916	0.867	0.865	0.834	0.817	0.766	0.984	0
$CE_{0.35;15;3}$	0.977	0.963	0.934	0.926	0.914	0.886	0.821	0.579	0
$CE_{0.40;15;3}$	0.971	0.948	0.910	0.907	0.889	0.866	0.819	0.689	0
$CE_{0.45;15;3}$	0.962	0.928	0.880	0.881	0.850	0.831	0.774	0.893	0
$CE_{0.35;20;3}$	0.981	0.968	0.941	0.935	0.922	0.895	0.827	0.529	100
$CE_{0.40;20;3}$	0.975	0.954	0.917	0.915	0.897	0.872	0.817	0.652	0
$CE_{0.45;20;3}$	0.966	0.934	0.887	0.889	0.857	0.837	0.771	0.859	0
$CE_{GC_p}$	0.981	0.968	0.941	0.935	0.922	0.895	0.827	0.529	—
$LR_{0.35}$	0.989	0.978	0.874	0.504	0.047	-1.102	-4.739	9.448	1
$LR_{0.40}$	0.989	0.983	0.921	0.718	0.600	0.383	-0.116	2.522	27
$LR_{0.45}$	0.989	0.983	0.912	0.681	0.587	0.627	0.241	1.980	72
$LR_{GC_p}$	0.990	0.985	0.921	0.725	0.612	0.492	-0.017	2.292	—
$SS_{15.2}$	0.998	0.991	0.950	0.857	0.683	-0.366	-10.013	12.901	0
$SS_{14.3}$	0.998	0.992	0.957	0.885	0.793	0.345	-3.210	5.239	4
$SS_{13.2}$	0.998	0.992	0.959	0.895	0.850	0.680	-0.470	2.096	96
$SS_{GC_p}$	0.998	0.992	0.959	0.891	0.838	0.621	-0.858	2.558	—

Table 2: Recovering derivatives of the oscillatory test function ( $n = 500$ ).

Strategy	0	1	2	3	4	5	6	RSD	$GC_p$ votes
$CE_{0.35;10;2}$	0.651	0.842	0.793	0.728	0.472	-0.274	-2.659	6.445	0
$CE_{0.40;10;2}$	0.941	0.915	0.881	0.854	0.787	0.563	-0.094	2.152	0
$CE_{0.45;10;2}$	0.945	0.908	0.867	0.844	0.782	0.674	0.328	1.653	0
$CE_{0.35;15;2}$	0.959	0.930	0.869	0.698	-0.011	-3.642	-24.817	32.015	0
$CE_{0.40;15;2}$	0.962	0.937	0.904	0.849	0.697	0.005	-3.129	5.775	0
$CE_{0.45;15;2}$	0.951	0.915	0.874	0.843	0.751	0.474	-0.623	2.814	0
$CE_{0.35;20;2}$	0.970	0.942	0.855	0.451	-2.326	-23.254	-190.543	219.905	0
$CE_{0.40;20;2}$	0.965	0.942	0.905	0.816	0.424	-2.038	-18.443	23.429	0
$CE_{0.45;20;2}$	0.952	0.917	0.874	0.830	0.673	-0.012	-3.666	6.433	0
$CE_{0.35;10;3}$	0.958	0.942	0.910	0.895	0.881	0.849	0.771	0.794	0
$CE_{0.40;10;3}$	0.958	0.935	0.896	0.889	0.873	0.853	0.812	0.785	0
$CE_{0.45;10;3}$	0.952	0.916	0.867	0.866	0.835	0.819	0.768	0.977	0
$CE_{0.35;15;3}$	0.977	0.961	0.930	0.924	0.911	0.889	0.837	0.570	0
$CE_{0.40;15;3}$	0.972	0.949	0.910	0.908	0.891	0.871	0.831	0.668	0
$CE_{0.45;15;3}$	0.963	0.929	0.881	0.882	0.851	0.834	0.778	0.882	0
$CE_{0.35;20;3}$	0.981	0.967	0.938	0.934	0.920	0.900	0.846	0.513	100
$CE_{0.40;20;3}$	0.976	0.954	0.917	0.916	0.899	0.877	0.829	0.631	0
$CE_{0.45;20;3}$	0.967	0.934	0.888	0.890	0.859	0.840	0.776	0.847	0
$CE_{GC_p}$	0.981	0.967	0.938	0.934	0.920	0.900	0.846	0.513	—
$LR_{0.35}$	0.991	0.992	0.968	0.859	0.717	0.488	-0.127	2.113	40
$LR_{0.40}$	0.990	0.991	0.967	0.850	0.741	0.746	0.481	1.233	31
$LR_{0.45}$	0.990	0.989	0.936	0.730	0.627	0.750	0.408	1.570	29
$LR_{GC_p}$	0.991	0.992	0.964	0.849	0.745	0.624	0.218	1.617	—
$SS_{15.2}$	0.999	0.997	0.982	0.934	0.866	0.595	-1.654	3.281	0
$SS_{14.3}$	0.999	0.997	0.980	0.932	0.885	0.753	-0.262	1.715	0
$SS_{13.2}$	0.999	0.996	0.974	0.920	0.891	0.836	0.203	1.180	100
$SS_{GC_p}$	0.999	0.996	0.974	0.920	0.891	0.836	0.203	1.180	—

Table 3: Recovering derivatives of the monotone test function ( $n = 125$ ).

Strategy	0	1	2	3	4	5	6	RSD	$GC_p$ votes
$CE_{0.35;10;2}$	0.996	0.951	0.303	-12.342	-5.45E+02	-5.41E+04	-1.37E+07	1.38E+07	0
$CE_{0.40;10;2}$	0.996	0.960	0.517	-6.690	-1.92E+02	-1.30E+04	-2.91E+06	2.92E+06	0
$CE_{0.45;10;2}$	0.996	0.962	0.542	-5.127	-1.03E+02	-6.29E+03	-1.14E+06	1.15E+06	7
$CE_{0.35;15;2}$	0.996	0.945	0.186	-14.265	-7.59E+02	-1.05E+05	-3.97E+07	3.98E+07	0
$CE_{0.40;15;2}$	0.996	0.958	0.477	-6.864	-2.17E+02	-1.95E+04	-7.01E+06	7.03E+06	0
$CE_{0.45;15;2}$	0.996	0.962	0.535	-5.067	-1.11E+02	-8.29E+03	-2.35E+06	2.36E+06	5
$CE_{0.35;20;2}$	0.996	0.943	0.137	-15.703	-9.97E+02	-1.77E+05	-9.31E+07	9.32E+07	15
$CE_{0.40;20;2}$	0.996	0.957	0.469	-6.953	-2.71E+02	-3.85E+04	-2.25E+07	2.25E+07	3
$CE_{0.45;20;2}$	0.996	0.962	0.532	-5.057	-1.27E+02	-1.39E+04	-6.83E+06	6.84E+06	36
$CE_{0.35;10;3}$	0.995	0.922	0.109	-11.090	-6.11E+02	-3.71E+04	-8.82E+06	8.86E+06	6
$CE_{0.40;10;3}$	0.996	0.948	0.507	-5.466	-1.99E+02	-1.01E+04	-2.33E+06	2.34E+06	3
$CE_{0.45;10;3}$	0.996	0.959	0.628	-3.826	-1.02E+02	-4.49E+03	-8.90E+05	8.94E+05	14
$CE_{0.35;15;3}$	0.995	0.921	0.110	-11.114	-5.82E+02	-3.90E+04	-1.08E+07	1.08E+07	5
$CE_{0.40;15;3}$	0.996	0.948	0.508	-5.623	-2.01E+02	-1.07E+04	-2.66E+06	2.67E+06	0
$CE_{0.45;15;3}$	0.996	0.958	0.624	-3.995	-1.04E+02	-4.78E+03	-9.80E+05	9.85E+05	2
$CE_{0.35;20;3}$	0.995	0.921	0.115	-10.819	-5.45E+02	-3.85E+04	-1.10E+07	1.11E+07	0
$CE_{0.40;20;3}$	0.996	0.948	0.506	-5.611	-1.97E+02	-1.08E+04	-2.75E+06	2.76E+06	2
$CE_{0.45;20;3}$	0.996	0.958	0.621	-4.028	-1.04E+02	-4.84E+03	-1.02E+06	1.03E+06	2
$CE_{GC_p}$	0.996	0.940	0.297	-9.675	-5.23E+02	-6.58E+04	-3.27E+07	3.28E+07	—
$LR_{0.35}$	0.988	0.405	-39.917	-1730.140	-5.67E+04	-4.74E+06	-2.61E+09	2.62E+09	11
$LR_{0.40}$	0.991	0.574	-19.613	-655.991	-1.47E+04	-1.79E+06	-5.09E+08	5.11E+08	7
$LR_{0.45}$	0.992	0.668	-11.695	-316.349	-6.63E+03	-9.10E+05	-1.47E+08	1.48E+08	82
$LR_{GC_p}$	0.991	0.596	-20.185	-728.027	-1.99E+04	-1.87E+06	-8.64E+08	8.66E+08	—
$SS_{15.2}$	0.993	0.621	-16.543	-502.917	-1.42E+04	-2.69E+06	-2.84E+09	2.84E+09	47
$SS_{14.3}$	0.993	0.678	-11.797	-311.702	-7.28E+03	-1.15E+06	-1.01E+09	1.02E+09	0
$SS_{13.2}$	0.993	0.734	-7.881	-178.595	-3.35E+03	-4.26E+05	-3.05E+08	3.05E+08	53
$SS_{GC_p}$	0.993	0.665	-13.387	-393.919	-1.10E+04	-2.00E+06	-1.79E+09	1.79E+09	—

Table 4: Recovering derivatives of the monotone test function ( $n = 500$ ).

Strategy	0	1	2	3	4	5	6	RSD	$GC_p$ votes
$CE_{0.35;10;2}$	0.998	0.984	0.632	-6.650	-2.34E+02	-2.91E+04	-7.22E+06	7.25E+06	0
$CE_{0.40;10;2}$	0.999	0.987	0.731	-4.328	-8.90E+01	-5.96E+03	-1.22E+06	1.22E+06	0
$CE_{0.45;10;2}$	0.999	0.985	0.693	-3.686	-5.28E+01	-3.26E+03	-5.09E+05	5.13E+05	8
$CE_{0.35;15;2}$	0.999	0.982	0.545	-7.892	-3.95E+02	-5.94E+04	-2.22E+07	2.22E+07	2
$CE_{0.40;15;2}$	0.999	0.985	0.692	-4.293	-9.43E+01	-8.54E+03	-3.20E+06	3.21E+06	0
$CE_{0.45;15;2}$	0.999	0.984	0.682	-3.582	-5.56E+01	-3.99E+03	-1.06E+06	1.06E+06	4
$CE_{0.35;20;2}$	0.999	0.981	0.510	-8.465	-5.04E+02	-8.41E+04	-3.94E+07	3.94E+07	10
$CE_{0.40;20;2}$	0.999	0.984	0.688	-4.154	-1.18E+02	-1.82E+04	-1.18E+07	1.18E+07	1
$CE_{0.45;20;2}$	0.999	0.984	0.678	-3.518	-6.33E+01	-6.74E+03	-3.44E+06	3.45E+06	16
$CE_{0.35;10;3}$	0.999	0.986	0.807	-3.625	-1.54E+02	-7.53E+03	-1.83E+06	1.84E+06	6
$CE_{0.40;10;3}$	0.999	0.989	0.834	-2.768	-8.53E+01	-3.35E+03	-7.88E+05	7.91E+05	6
$CE_{0.45;10;3}$	0.999	0.990	0.840	-2.266	-5.15E+01	-1.79E+03	-3.50E+05	3.51E+05	23
$CE_{0.35;15;3}$	0.999	0.986	0.804	-3.658	-1.45E+02	-8.01E+03	-2.18E+06	2.18E+06	3
$CE_{0.40;15;3}$	0.999	0.989	0.829	-2.904	-8.50E+01	-3.61E+03	-8.81E+05	8.85E+05	1
$CE_{0.45;15;3}$	0.999	0.990	0.834	-2.415	-5.26E+01	-1.94E+03	-3.85E+05	3.87E+05	9
$CE_{0.35;20;3}$	0.999	0.986	0.802	-3.577	-1.35E+02	-7.98E+03	-2.20E+06	2.21E+06	0
$CE_{0.40;20;3}$	0.999	0.989	0.826	-2.903	-8.28E+01	-3.62E+03	-8.96E+05	9.00E+05	3
$CE_{0.45;20;3}$	0.999	0.990	0.831	-2.448	-5.23E+01	-1.97E+03	-4.01E+05	4.03E+05	8
$CE_{GC_p}$	0.999	0.985	0.737	-3.946	-1.65E+02	-1.72E+04	-7.53E+06	7.55E+06	—
$LR_{0.35}$	0.997	0.870	-8.273	-383.427	-1.19E+04	-1.17E+06	-5.86E+08	5.87E+08	9
$LR_{0.40}$	0.998	0.907	-3.743	-155.854	-3.77E+03	-5.46E+05	-1.41E+08	1.41E+08	4
$LR_{0.45}$	0.998	0.932	-1.710	-70.351	-1.73E+03	-2.53E+05	-3.78E+07	3.81E+07	87
$LR_{GC_p}$	0.998	0.918	-3.334	-149.377	-4.22E+03	-4.98E+05	-1.88E+08	1.89E+08	—
$SS_{15.2}$	0.998	0.917	-3.250	-128.808	-3.59E+03	-6.46E+05	-5.53E+08	5.53E+08	7
$SS_{14.3}$	0.998	0.932	-1.988	-77.332	-1.81E+03	-2.73E+05	-2.02E+08	2.02E+08	1
$SS_{13.2}$	0.999	0.946	-0.984	-42.489	-8.20E+02	-1.00E+05	-6.44E+07	6.45E+07	92
$SS_{GC_p}$	0.998	0.941	-1.461	-62.683	-1.51E+03	-2.26E+05	-1.55E+08	1.56E+08	—